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On the Decay of the η -Meson.

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Summary. — The decay of the η -meson is estimated by considering higher mass contributions beside the ρ and ω . It is shown that the branching ratio $\Gamma(\eta \rightarrow 2\gamma)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ comes out well while we get a little larger ratio for $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)/\Gamma(\eta \rightarrow 2\gamma)$, although it may not be much in disagreement with present data.

1. — Since the discovery of the spin-parity 1^- octet, many phenomena have been explained by the use of pole diagrams due to these particles. Particularly the branching ratio

$$\Gamma(\eta \rightarrow 2\gamma)/\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)$$

for 0^{+-} η -meson was estimated by GELL-MANN *et al.* (1) provided that these decays go through a ρ pole diagram.

Nevertheless the application of the same idea to $\eta \rightarrow 3\pi$ decay met with difficulty due to the closed-loop integration. Beside the divergence problem, there seems to remain the difficulty of getting the observed ratio ~ 3 for $\Gamma(\eta \rightarrow \text{neutral})/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ because of the smallness of the three-body phase volume.

Recently, however, a mechanism which leads to suppression for $\pi^0 \rightarrow 2\gamma$ decay was analyzed by GEFEN (2). He obtains this result by taking into account higher mass contributions, beside those of ρ and ω , as suggested by

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(1) M. GELL-MANN, D. SHARP and W. G. WAGNER: *Phys. Rev. Lett.*, **8**, 261 (1962); S. HORI, S. ONEDA, S. CHIBA and H. HIRAKI: *Phys. Lett.*, **1**, 81 (1962).

(2) D. A. GEFFEN: *Phys. Rev.*, **128**, 374 (1962).

the experimental form factor of $\pi^0 \rightarrow \gamma e^+ e^-$. According to the idea of unitary symmetry, then, one would expect the same cancellation for $\eta \rightarrow 2\gamma$ decay. This fact indicates that the estimate of the ratio $\Gamma(\eta \rightarrow 2\gamma)/\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0)$, based on the simple phase volume arguments, is no longer adequate. To make it clear, we have to see how the same mechanism works on the $\eta \rightarrow 3\pi$ decay.

In this regard, it was pointed out by several people⁽³⁾ that there is a similarity of the Dalitz plots for τ' and η -decays. A reasonable explanation was given by saying that these decay modes are intermediated by the following steps, $K^+ \rightarrow \pi^+ \rightarrow \pi^0 \pi^0 \pi^+$ and $\eta \rightarrow \pi^0 \rightarrow \pi^+ \pi^- \pi^0$. Of these steps the first one is a $\Delta I = \frac{1}{2}$ transition for K^+ and $\Delta I = 1$ e.m. transition for $0^{-+} \eta$. Then the isobaric spin of the final state is $I = 1$ for both cases and, as the second step is due to the strong interaction, the Dalitz plots will be expected to be similar.

Based on this argument, we shall assume that the η - π^0 transition is the dominant contribution for $\eta \rightarrow 3\pi$ decay. As we will see later, this assumption leads to such a unique choice that we can conclude that the higher mass states, mentioned, should behave as a vector singlet state.

Several consequences which follow from the η - π transition mechanism, together with the contribution from the higher vector singlet state, are examined in this paper.

2. - Let us define the vertex function between the P.S. state, a , and two vector states, b , c , as follows,

$$(1) \quad \Gamma_{\mu\nu}^{abc}(k, k') \equiv \langle c_\nu(k') | j_\mu^{(b)}(0) | a(k+k') \rangle = \varepsilon_{\mu\nu\lambda\tau} k_\lambda k'_\tau F^{abc}(k^2)/m_\pi,$$

where $j_\mu^{(b)}$ is the current for the b -meson. The γ_ρ and γ_ω are used hereafter as the renormalized coupling constants which appear in the currents, $j_\mu^{(\rho)}$, $j_\mu^{(\omega)}$ in a similar way as e does in $j_\mu^{(\gamma)}$ ⁽⁴⁾.

If the ρ , ω contribution is taken to be important the form factor for $\pi^0 \rightarrow \gamma e^+ e^-$ is given by the formula

$$(2) \quad F^{\pi^0\gamma\gamma}(k^2) = f_{\pi^0\rho\omega} \frac{e^2}{2\sqrt{3}} \frac{1}{\gamma_\rho\gamma_\omega} \left(\frac{m_\rho^2}{m_\rho^2 + k^2} + \xi \right),$$

where we put $F^{\pi^0\rho\omega}(k^2) = F^{\pi^0\omega\rho}(k^2) = f_{\pi^0\rho\omega}$ and $m_\rho = m_\omega$. A parameter ξ is inserted by GEFEN⁽²⁾ in order to take into account the higher mass contributions.

⁽³⁾ G. BARTON and S. P. ROSEN: *Phys. Rev. Lett.*, **8**, 414 (1962); M. A. BAQI BÉG: *Phys. Rev. Lett.*, **9**, 67 (1962).

⁽⁴⁾ M. GELL-MANN and F. ZACHARIASEN: *Phys. Rev.*, **124**, 953 (1961).

Instead of eq. (2), however, let us assume in this paper that the higher mass contributions can be represented by a state with a mass m_x . Since this state transforms into a γ , it may be natural to take it as a vector state. According to the unitary symmetry, we have several sets of states, namely the vector singlet and the vector octet etc.

As we will see later, the vector octet intermediate states however, give no η - π transition in the limit of unitary symmetry. Thus, confining ourselves to the lower configuration, the representative state for the higher mass contribution should be taken as a vector singlet, if we insist on the importance of the η - π transition.

In this case, $F^{\pi^0\gamma\gamma}(k^2)$ is defined, instead of by eq. (2), by

$$\begin{aligned}
 (3) \quad F^{\pi^0\gamma\gamma}(k^2) &= e^2 \left(\frac{f_{\pi\rho\omega}}{2\sqrt{3}\gamma_\omega} + \frac{f_{\pi\rho x}}{2\sqrt{3}\gamma_x} \right) \left(\frac{m_\rho^2}{m_\rho^2 + k^2} \frac{1}{2\gamma_\rho} \right) + \\
 &\quad + \frac{1}{2\gamma_\rho} \left(\frac{f_{\pi\rho\omega}}{2\sqrt{3}\gamma_\omega} \frac{m_\omega^2}{m_\omega^2 + k^2} + \frac{f_{\pi\rho x}}{2\sqrt{3}\gamma_x} \frac{m_x^2}{m_x^2 + k^2} \right) = \\
 &= \frac{f_{\pi\omega\rho} Q^2}{4\sqrt{3}\gamma_\omega\gamma_\rho} \left[(2 + \xi) \frac{m_\rho^2}{m_\rho^2 + k^2} + \xi \frac{m_x^2}{m_x^2 + k^2} \right],
 \end{aligned}$$

where

$$(4) \quad \xi \equiv \left(\frac{f_{\pi\rho x}}{\gamma_x} \right) \left(\frac{f_{\pi\rho\omega}}{\gamma_\omega} \right)^{-1}.$$

Comparing eq. (3) with the experimental form factor for $\pi^0 \rightarrow \gamma e^+ e^-$,

$$\begin{aligned}
 (5) \quad F_{\text{exp}}^{\pi^0\gamma\gamma}(k^2) &= F^{\pi^0\gamma\gamma}(0) [1 + ak^2/m_\pi^2], \\
 a &= -0.24 \pm 0.16,
 \end{aligned}$$

we get

$$(6) \quad \xi = -1.12 \quad \text{for} \quad m_x \sim 2m_\rho \quad \text{and} \quad a = -0.1.$$

The smaller m_x and the larger $|a|$ will bring ξ to be very near to -1 which will not be consistent with the $\omega \rightarrow 3\pi$ decay rate (*).

The $\pi^0 \rightarrow 2\gamma$ decay rate is given by

$$(7) \quad \Gamma(\pi^0 \rightarrow 2\gamma) = \frac{[F^{\pi^0\gamma\gamma}(0)]^2}{64\pi} m_\pi \sim 3 \cdot 10^{-6} \text{ MeV},$$

(*) Our choice eq. (6) gives $\Gamma(\omega \rightarrow 3\pi) = 9.4 \text{ MeV}$.

and it follows from eq. (7) that

$$(8) \quad F^{\pi^0\gamma\gamma}(0) = (4\pi \cdot 0.7 \cdot 10^{-2})^{\frac{1}{2}} \alpha.$$

Comparing eq. (8) with eq. (3), we can estimate $f_{\pi\rho\omega}$ as

$$(9) \quad \frac{f_{\pi\rho\omega}^2}{4\sqrt{3}\gamma_\rho\gamma_\omega} = \frac{F^{\pi^0\gamma\gamma}(0)}{\alpha} \frac{\gamma_\rho^2\gamma_\omega^2/(4\pi)^2}{4\sqrt{3}\gamma_\rho\gamma_\omega} (2\sqrt{3})^2 \left(\frac{1}{1+\xi}\right)^2 = 0.43,$$

where we put $\gamma_\rho^2/4\pi \simeq \gamma_\omega^2/4\pi \simeq \frac{1}{2}$. The $f_{\pi\rho\omega}$ calculated in this way is nine times larger than that calculated by GELL-MANN *et al.* (1) in the case $\xi=0$.

It is easy to see at this point that the experiment (5) suggests really the occurrence of the cancellation between contributions from ρ , ω and \mathbf{x} . Then the unitary symmetry (5) predicts the same cancellation for $\eta \rightarrow 2\gamma$.

Namely we have

$$(10) \quad F^{\eta\gamma\gamma}(0) = \frac{2e^2 f_{\eta\rho\rho}}{4\gamma_\rho^2} \left[1 - \frac{1}{3} (1 + 2\xi') \right],$$

where

$$2\xi' \equiv \left(\frac{f_{\eta\omega\mathbf{x}}}{\gamma_{\mathbf{x}}} \right) \left(\frac{f_{\eta\omega\omega}}{\gamma_\omega} \right)^{-1}.$$

Since the unitary symmetry gives the equalities

$$(11) \quad f_{\eta\rho\mathbf{x}} = f_{\eta\omega\mathbf{x}} = f_{\pi\rho\mathbf{x}} \quad \text{and} \quad f_{\eta\omega\omega} = -f_{\eta\rho\rho} = -\frac{1}{2}f_{\pi\rho\omega},$$

we get in this limit $\xi' = -\xi$.

The decay rate, $\Gamma(\eta \rightarrow 2\gamma)$, depends on the value ξ' strongly. Thus we shall estimate it for two cases, one for $\xi' = -\xi$ and the other for $\xi' = 1.5$ which means that we consider a small deviation from eq. (11). Then we have

$$(12) \quad \left\{ \begin{array}{l} \Gamma(\eta \rightarrow 2\gamma) = \left(\frac{\sqrt{3}}{2}\right)^2 \left[\frac{1 - \frac{1}{3}(1 + 2\xi')}{1 + \xi} \right]^2 \left(\frac{m_\eta}{m_\pi}\right)^3 \Gamma(\pi_0 \rightarrow 2\gamma) = \\ \quad = 0.64 \cdot 10^{-4} \text{ MeV} \quad \text{for } \xi' = -\xi = 1.12, \\ \quad = 10.8 \cdot 10^{-4} \text{ MeV} \quad \text{for } \xi' = 1.5. \end{array} \right.$$

(5) M. GELL-MANN: *Phys. Rev.*, **125**, 1067 (1962); M. IKEDA, S. OGAWA and Y. OHNUKI: *Suppl. of Progr. Theor. Phys.*, **19**, 44 (1961); N. CABIBBO and R. GATTO: *Nuovo Cimento*, **21**, 872 (1961).

3. - The diagrams for $\eta \rightarrow 3\pi$ decay through ρ , ω intermediate states are given in Fig. 1 where the loops contain $\rho\gamma$ and $\omega\gamma$ e.m. transitions as is illustrated in Fig. 1a.

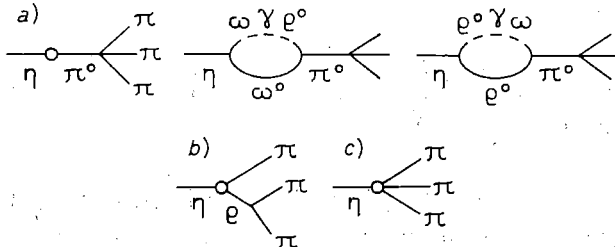


Fig. 1.

Based on the Dalitz plot argument, as we discussed in the introduction, we simply disregard the diagrams in Fig. 1b and Fig. 1c. But this is not valid if we consider only contributions from ρ , ω , because the diagrams in Fig. 1a are zero in the limit of unitary symmetry. This is easily seen from the relation in eq. (11).

The situation remains the same even if we consider the contribution to $\eta\text{-}\pi^0$ transition from other vector octet states.

Therefore the assumption of the $\eta\text{-}\pi^0$ transition to be dominant leads to the unique choice that the higher mass contribution belongs to a vector singlet.

Now the $\eta\text{-}\pi^0$ transition through ρ , ω and this intermediate states is given by

$$(13) \quad M_{\eta\pi}(q)^2 = \frac{e^2 f_{\pi\rho\omega}^2}{4\sqrt{3}\gamma_\omega\gamma_\rho} \left[(-\xi' + \xi) I_1(q^2) + (-\xi'\xi) \left(\frac{\gamma_x}{\gamma_\omega} \right)^2 I_2(q^2) \right] / m_\pi^2,$$

where

$$I_1(q^2) = \frac{4}{(2\pi)^4} \int \frac{m_\rho^2}{m_\rho^2 + k^2} \frac{m_x^2}{m_x^2 + k^2} \frac{1}{k^2} \frac{k^2 q^2 - (kq)^2}{m_\rho^2 + (q-k)^2} d^4k,$$

and

$$I_2(q^2) = \frac{4}{(2\pi)^4} \int \left(\frac{m_\rho^2}{m_\rho^2 + k^2} \right)^2 \frac{1}{k^2} \frac{k^2 q^2 - (kq)^2}{m_x^2 + (q-k)^2} d^4k.$$

These are convergent integrations and thus we have the following formulas which are valid for $0 < -q^2 \lesssim m_\eta^2$:

$$(14) \quad \begin{cases} I_1(q^2) = \frac{1}{(4\pi)^2} \frac{1}{6} \frac{3}{4} q^2 \frac{m_x^2}{3}, \\ I_2(q^2) = \frac{1}{(4\pi)^2} \frac{1}{6} \frac{3}{4} q^2 \frac{m_\rho^2}{4}. \end{cases}$$

We see here that the ratio of the second term with respect to the first in eq. (13) is really small, for $m_x = 2m_\rho$ and $\xi' = -\xi = -1.12$, that is, $\sim \frac{1}{16} (\gamma_x/\gamma_\omega)^2$. Although we do not know the value γ_x , it may not differ much from γ_ω and then we may disregard the second term in eq. (13).

Thus we get the η - π transition coefficient

$$(15) \quad l_{\eta\pi} \equiv M_{\eta\pi}(q^2) \frac{1}{q^2 + m_\pi^2} \Big|_{q^2 = -m_\eta^2} = \frac{\alpha}{\pi} 0.86(\xi' - \xi) = 0.61\alpha \text{ for } \xi' = -\xi = 1.12, \\ = 0.7\alpha \text{ for } \xi' = 1.5, \xi = -1.12.$$

Here let us define an effective Hamiltonian density for the S -wave π - π scattering by eq. (16),

$$(16) \quad H_{\pi\pi} = \frac{d}{8} (\pi\pi)(\pi\pi).$$

Then the decay rate for $\eta \rightarrow \pi^+\pi^-\pi^0$ is

$$(17) \quad \Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) = \left(\frac{\sqrt{6}d}{4\pi}\right)^2 l_{\eta\pi}^2 0.020m_\pi.$$

Unfortunately we are not in a position to have a definite value of the scattering length from $T=0$, S -wave π - π scattering. So we shall estimate $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ for several choices of the scattering length, *i.e.*, $a_{s_0} = 0.5$ ⁽⁶⁾, 1, 1.5 ⁽⁷⁾ m_π^{-1} . These correspond to the following values of the effective coupling constants: $d/4\pi = 0.4, 0.8, 1.2$ ^(*).

TABLE I.

	$\xi' = -\xi = 1.12$			$\xi' = 1.5, \xi = -1.12$		
	0.4 (0.5)	0.8 (1)	1.2 (1.5)	0.4 (0.5)	0.8 (1)	1.2 (1.5)
$-d/4\pi (m_\pi A_{s_0})$						
$\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ in 10^{-4} MeV	0.53	2.13	4.8	0.7	2.9	6.5
γ_1 ^(*) $\equiv \frac{\Gamma(\eta \rightarrow 2\gamma)}{\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)}$	1.2	0.3	0.13	15.0	3.7	1.7

(*) The linear dependence of the $\eta \rightarrow 3\pi$ matrix element on y , which is an experimental fact for the Dalitz plot of the $\eta \rightarrow 3\pi$, tells us that the ratio $\Gamma(\eta \rightarrow \pi^0\pi^0\pi^0)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ is 1.5 in the nonrelativistic limit.

⁽⁶⁾ H. J. SCHNITZER: *Phys. Rev.*, **125**, 1059 (1962).

⁽⁷⁾ B. R. DESAI: *Phys. Rev. Lett.*, **6**, 497 (1961); N. BOOTH, A. ABASHIAN and K. M. CROWE: *Phys. Rev. Lett.*, **7**, 35 (1961).

(*) In the lowest order perturbation approximation, $d/4\pi = 4\lambda$ where λ is given by CHEW and MANDELSTAM ⁽⁸⁾.

⁽⁸⁾ G. F. CHEW and S. MANDELSTAM: *Phys. Rev.*, **119**, 467 (1960).

The decay rate for $\eta \rightarrow \pi^+\pi^-\pi^0$ and the branching ratio $\Gamma(\eta \rightarrow 2\gamma)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ estimated in this way are given in the Table I.

The evaluation of the decay rate $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)$ is carried out through a ρ -meson pole diagram. As the vector singlet state x gives no contribution to this process, we get the ratio

$$(18) \quad \gamma_2 \equiv \frac{\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)}{\Gamma(\eta \rightarrow 2\gamma)} = \frac{\gamma_\rho^2}{4\pi} \frac{\gamma_{\rho\pi\pi}^2}{4\pi} \frac{1}{\alpha} \frac{10 \cdot 10^{-3}}{[1 - \frac{1}{3}(1 + 2\xi')]^2}.$$

For $\xi' = 1.12$ and $\xi' = 1.5$ with $\gamma_\rho^2/4\pi \sim \gamma_{\rho\pi\pi}^2/4\pi \sim \frac{1}{2}$, γ_2 takes the values 5.3 and 0.3, respectively. Thus we see that the case of exact unitary symmetry is not consistent with the observed data⁽⁹⁾ whereas the second case, with small deviation from unitary symmetry, is not necessarily inconsistent.

4. - We started with the assumption that the vector particle γ -transition is important whenever the e.m. interaction comes in. The observed form factor for $\pi^0 \rightarrow \gamma e^+e^-$, then, tells us that there should be some contribution from the higher mass states besides those from ρ and ω . We represented them by vector particle states. Here we have two choices for taking them as the octet states and as the singlet state.

Both cases lead to the cancellation for the $\pi^0 \rightarrow 2\gamma$ amplitude and then the $\eta \rightarrow 2\gamma$ suppression follows from this. This gives the possibility of explaining the large discrepancy for the branching ratio γ_1 between the simple calculation and the experiment.

A difference, however, appears between octet states and a singlet state, if we consider that η - π and the $\eta \rightarrow \pi^+\pi^-\gamma$ transitions. The vector octet gives no η - π transition and suppresses $\eta \rightarrow \pi^+\pi^-\gamma$ while the vector singlet leads to a large η - π transition coefficient and to no cancellation for $\eta \rightarrow \pi^+\pi^-\gamma$.

We made an assumption that the η - π transition is important. In this way we can examine if this assumption is consistent with other observed data, as well as the validity of a choice for a vector singlet state.

As a matter of fact, the results which we get in this case depend on the value of the scattering length for the S -wave π - π scattering. For example, if we consider a small deviation from unitary symmetry and take $\xi' = 1.5$, $\xi = 1.12$, the branching ratios γ_1 , γ_2 , given in the Table I and by eq. (18) for $a_{s_0} = 1.5 m_\pi^{-1}$, may not be in disagreement with the experiment.

But if a_{s_0} turns out to be as small as $1 m_\pi^{-1}$ or the experimental value for γ_2 settles smaller than 0.1, it seems difficult to explain both γ_1 and γ_2 by this model. In other words we have to give up with the η - π transition mechanism as well as with the choice of the vector singlet.

(⁹) P. L. BASTIEN, J. P. BERGE, O. I. DAHL, M. FERRO-LUZZI, D. H. MILLER, J. J. MURRAY, A. H. ROSENFELD and M. B. WATSON: *Phys. Rev. Lett.*, **8**, 114 (1962).

The other test for this model will come from the decay rate for $\omega \rightarrow 3\pi$. With a vector singlet, as it does not contribute to this process, we get $\Gamma(\omega \rightarrow 3\pi) \sim 10$ MeV. While with the vector octet, where a cancellation happens, we have still the possibility of getting a smaller value for this.

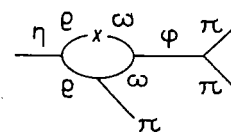


Fig. 2.

As regards the η - 3π decay in this case the alternative to the η - π transition will be the diagram shown in Fig. 2, although we have to encounter an unknown coupling $f_{\varphi\rho\rho}$, where φ is a $T=0$, 0^{++} particle⁽¹⁰⁾, and the result is more ambiguous.

⁽¹⁰⁾ N. P. SAMIOS, A. H. BACHMANN, R. M. LEA, T. E. KALOGEROPOULOS and W. D. SHEPHARD: *Phys. Rev. Lett.*, **9**, 139 (1962).

Note added in proof.

In this paper we used the formula eq. (2) and eq. (3) etc. given by GELL-MANN *et al.*⁽⁴⁾ based on the assumption that the ρN interaction is R -invariant. But in order to have the γ - x transition, it is also essential that R -invariance is violated by some strong interactions.

RIASSUNTO (*)

Si valuta il decadimento del mesone η prendendo in considerazione contributi di masse maggiori oltre ad ρ ed ω . Si dimostra che il rapporto di suddivisione $\Gamma(\eta \rightarrow 2\gamma)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ risulta abbastanza esatto mentre otteniamo un rapporto un po' più elevato per $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)/\Gamma(\eta \rightarrow 2\gamma)$, perquanto possa non essere molto in disaccordo con i dati attualmente disponibili.

(*) Traduzione a cura della Redazione.